Supply Chain Management Using Model Predictive Control

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Outline

• Supply Chain Management: Problems and Issues
• Model Predictive Control (MPC) Applied to Supply Chain Systems
• Case Studies: Two-Node Supply Chain
• Conclusions
• Future Research

Supply Chain Objectives
Economically supply desired products to the customer in the right amounts, when needed.
Issues:
• Determine what product(s) the customer desires
• Determine the demand of the product(s)
• Determine when the demand must be supplied
• Determine the best way to optimize the system to achieve desired profit margins

Supply chains are dynamical systems, and these issues merit a control-oriented approach

A control-oriented methodology for managing supply chains must have the following characteristics:

• Must Be Forward-Looking
• Make Use of Optimization
• Be Able to Handle Constraints
• Be Able to Track and Correct for Supply Time Errors
• Be Able to Track and Correct for Forecast Errors
• Be Robust (handle uncertainty)
Model Predictive Control (with enhancements) can serve as both an optimizer and a controller, meeting the functional requirements previously discussed.

Applying MPC to SCM

Goal: to show a proof-of-concept for a control-oriented approach to Supply Chain Systems

- Two-Node Supply Chain System (Manufacturer – Retailer)
- Information Sharing between the Manufacturer node and the Retailer node (Order Profile)
- Lead-Time agreement
Applying MPC to SCM

Manufacturer

Retailer

Retailer’s Model-Predictive Controller

O(k)

R(k)

A

R(k)

P(k)

S(k)

M

S(k)

M

I

M(k)

I

R(k)

S(k)

R

D(k)

Demand History Database

Forecaster

Order

Processing

Order

Shipping

Applying MPC to SCM

Manufacturer’s Policy

• Makes use of an Order Queue
  (First-come, first-serve basis)
• Partial shipping
  (Meet part of order, backorder rest)
• Multiple shipping
  (Meet as many orders as possible)
Manufacturer’s Policy

Manufacturer’s Supply Policy

- First order in queue is compared to inventory at hand. If there is not enough inventory the policy tries to use safety stock, if still not enough then the order is only partially met. The unmet amount is backordered.

If there is enough inventory to start with, the order is met and the policy goes to the next order in queue until there is not enough inventory.

Manufacturer’s Model for MPC

Retailer’s Policy

The retailer uses the following material balance:

\[ I_R(k+1) = I_R(k) + R_d(k) - S_R(k) \]

Where,

- \( I_R(k+1) \) ⇒ Retailer’s inventory level for next period
- \( I_R(k) \) ⇒ Retailer’s inventory level of current period
- \( R_d(k) \) ⇒ Retailer’s replenishment for current period
- \( S_R(k) \) ⇒ Retailer’s supply for current period
Retailer’s Model for MPC

\[ I_0(k + 1) = I_0(k) + R_0(k) - S_0(k) \]

Where, \( R_0 \) is related to \( R \) and the lead time as follows:

\[
\begin{align*}
R_0(k) &= S_0(k - \theta y) \\
S_0(k) &= O(k - \theta y) \\
O(k) &= R(k - \theta y)
\end{align*}
\]

Combining these relations one obtains the following:

\[
\begin{align*}
R_0(k) &= R(k - \theta y_0 - \theta y) \\
R_0(k) &= R(k - \tau)
\end{align*}
\]

Where, \( \tau \) is the leadtime and equals \( \theta y_0 + \theta y + \theta y \). Replacing this result into the model, we obtain:

\[ I_0(k + 1) = I_0(k) + R(k - \tau) - S_0(k) \]

In the s-domain this model becomes

\[ I_0(s) = \frac{1}{s - 1}R(s) - \frac{1}{s}S_0(s) \]

Demand Forecaster

The Forecaster uses an ARIMA model and past demand data to make predictions into the future. The ARIMA model in general can be expressed as follows:

\[ \Phi(z^{-1})\Delta^r z^{-1} z_o = \Theta(z^{-1}) w \]

Where,

\[
\begin{align*}
\Phi(z^{-1}) &= 1 - \phi_1 z^{-1} - \phi_2 z^{-2} - \ldots - \phi_p z^{-p} \\
\Theta(z^{-1}) &= 1 - \theta_1 z^{-1} - \theta_2 z^{-2} - \ldots - \theta_q z^{-q} \\
\Delta^r z^{-1} &= (1 - z^{-1})^r \\
\end{align*}
\]

\( z^{-1} \) = Backslash operator
\( \Phi(z^{-1}) \) = Autoregressive model
\( \Theta(z^{-1}) \) = Moving average model
\( \Delta^r z^{-1} \) = Integrating model

The forecasts are calculated explicitly for \( l = 1, 2, \ldots \), recursively as follows:

\[ e(l) = \sum_{j=1}^{\infty} \phi_j e(l - j) - \sum_{j=1}^{\infty} \theta_j e(l+j) \]

Economics

The manufacturer economics through costs and revenue are calculated using the following formulas:

Cost:

\[ K_u(k) = A + C_u P(k) + \sum_{i} C_{u,w}(i) I_0(k + 1) + I_0(k) + \sum_{i} C_{u,b}(i) R_0(k) + A \]

Profit:

\[ \Phi_u(k) = \pi_u R_0(k) - K_u(k) \]

The retailers economics can be calculated using the following relations:

Cost:

\[ K_0(k) = A_0 + C_0 R_0(k) + \sum_{i} C_{0,w}(i) I_0(k + 1) + I_0(k) + \sum_{i} C_{0,b}(i) R_0(k) + A_0 \]

Profit:

\[ \Phi(k) = \pi_0 R_0(k) - K_0(k) \]

<table>
<thead>
<tr>
<th>Manufacturer’s Costs</th>
<th>Retailer’s Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production/Unit Cost:</td>
<td>2</td>
</tr>
<tr>
<td>Cost Per Unit Shipped:</td>
<td>6</td>
</tr>
<tr>
<td>Cost Per Unit Backordered:</td>
<td>4</td>
</tr>
<tr>
<td>Storage Per Unit Cost:</td>
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<tr>
<td>Carrying Cost Rate:</td>
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<tr>
<td>Sales Per Unit Cost:</td>
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<tr>
<td>Order Processing Cost:</td>
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<tr>
<td>Cost Per Unit Ordered:</td>
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<tr>
<td>Cost Per Unit Shipped:</td>
<td>6</td>
</tr>
<tr>
<td>Cost Per Unit Backordered:</td>
<td>4</td>
</tr>
<tr>
<td>Storage Per Unit Cost:</td>
<td>2</td>
</tr>
<tr>
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<td>Shipping Cost:</td>
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</tr>
<tr>
<td>Sales Per Unit Cost:</td>
<td>15</td>
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Case 1a: Deterministic Demand

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<tr>
<th>Demand</th>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Manufacturer’s Settings**
- Initial Inventory: 0 units
- Predictive Horizon: 10 periods
- Moves: 5
- Input Weight: 0
- Moves-Weight: 0
- Output Weight: 1
- Input Limits: Min=0, Max=inf
- Moves Limits: Min=inf, Max=inf
- Output Limits: Min=0, Max=inf
- Safety Stock: 700 units
- Production Lead Time: 3 periods
- Ordering Lead Time: 1 period
- Transportation Lead Time: 1 period
- Forecast Flag: No
- Partial Shipping: Yes
- Anticipation Flag: Yes

**Retailer’s Settings**
- Initial Inventory: 0 units
- Predictive Horizon: 15 periods
- Moves: 10
- Input Weight: 0
- Moves-Weight: 0
- Output Weight: 1
- Input Limits: Min=0, Max=inf
- Moves Limits: Min=inf, Max=inf
- Output Limits: Min=0, Max=inf
- Safety Stock: 200 units
- Production Lead Time: 1 period
- Ordering Lead Time: 1 period
- Transportation Lead Time: 1 period
- Forecast Flag: Yes
- Partial Shipping: Yes
- Anticipation Flag: Yes
Case 1a: Deterministic Demand

Case 1a: Net Costs

Case 1b: Move Suppression

<table>
<thead>
<tr>
<th>Manufacturer's Settings</th>
<th>Values</th>
<th>Retailer's Settings</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Inventory</td>
<td>0 units</td>
<td>Initial Inventory</td>
<td>0 units</td>
</tr>
<tr>
<td>Predictive Horizon</td>
<td>10</td>
<td>Predictive Horizon</td>
<td>15</td>
</tr>
<tr>
<td>Moves</td>
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<td>Moves</td>
<td>10</td>
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<tr>
<td>Input Weight</td>
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<td>Input Weight</td>
<td>0</td>
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<tr>
<td>Moves Weight</td>
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<td>Moves Weight</td>
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<tr>
<td>Output Weight</td>
<td>1</td>
<td>Output Weight</td>
<td>1</td>
</tr>
<tr>
<td>Input Limits</td>
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<td>Input Limits</td>
<td>Min=0; Max=inf</td>
</tr>
<tr>
<td>Moves Limits</td>
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<td>Moves Limits</td>
<td>Min=-inf; Max=inf</td>
</tr>
<tr>
<td>Output Limits</td>
<td>Min=0; Max=inf</td>
<td>Output Limits</td>
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</tr>
<tr>
<td>Safety Stock</td>
<td>700 units</td>
<td>Safety Stock</td>
<td>700 units</td>
</tr>
<tr>
<td>Production Lead Time</td>
<td>3 periods</td>
<td>Forecast Flag</td>
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</tr>
<tr>
<td>Ordering Lead Time</td>
<td>1 period</td>
<td>Partial Shipping</td>
<td>Yes</td>
</tr>
<tr>
<td>Transportation Lead Time</td>
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<td>Anticipation Flag</td>
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</table>
### Case 1b: Move Suppression

<table>
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<tr>
<th>Manufacturer's Settings</th>
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<th>Values</th>
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<tbody>
<tr>
<td>Initial Inventory</td>
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<td>Initial Inventory</td>
<td>0 units</td>
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<tr>
<td>Predictive Horizon</td>
<td>10</td>
<td>Predictive Horizon</td>
<td>15</td>
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<tr>
<td>Moves</td>
<td>5</td>
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</tr>
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<td>Output Weight</td>
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<td>Output Weight</td>
<td>1</td>
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<tr>
<td>Input Limits</td>
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<td>Input Limits</td>
<td>Min=0, Max=inf</td>
</tr>
<tr>
<td>Moves Limits</td>
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<td>Min=-inf, Max=inf</td>
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<tr>
<td>Output Limits</td>
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<td>Output Limits</td>
<td>Min=0, Max=inf</td>
</tr>
<tr>
<td>Safety Stock</td>
<td>200 units</td>
<td>Safety Stock</td>
<td>200 units</td>
</tr>
<tr>
<td>Production Lead Time</td>
<td>6 periods</td>
<td>Forecast Flag</td>
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<tr>
<td>Ordering Lead Time</td>
<td>1 period</td>
<td>Partial Shipping Flag</td>
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<tr>
<td>Transportation Lead Time</td>
<td>1 period</td>
<td>Anticipation Flag</td>
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</table>

### Case 1c: Plant-Model Mismatch

**Case 1c: Plant Model Mismatch**

(No move suppression)
Case 1c: Plant-Model Mismatch (With Move Suppression)

Case 2: Stochastic Demand

Case 2: Stochastic Demand

<table>
<thead>
<tr>
<th>Manufacturer's Settings</th>
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<th>Retailer's Settings</th>
<th>Values</th>
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<tbody>
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<td>Initial Inventory</td>
<td>0 units</td>
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<td>Predictive Horizon</td>
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<td>Input Weight</td>
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<td>Input Weight</td>
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<tr>
<td>Moves Weight</td>
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<td>Moves Weight</td>
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<tr>
<td>Output Weight</td>
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<td>Output Weight</td>
<td>1</td>
</tr>
<tr>
<td>Input Limits</td>
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<td>Input Limits</td>
<td>Min= -inf; Max= inf</td>
</tr>
<tr>
<td>Moves Limits</td>
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<td>Moves Limits</td>
<td>Min= -inf; Max= inf</td>
</tr>
<tr>
<td>Output Limits</td>
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<td>Output Limits</td>
<td>Min= 0; Max= inf</td>
</tr>
<tr>
<td>Safety Stock</td>
<td>200 units</td>
<td>Safety Stock</td>
<td>200 units</td>
</tr>
<tr>
<td>Production Lead Time</td>
<td>3 periods</td>
<td>Forecast Flag</td>
<td>Yes</td>
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<tr>
<td>Ordering Lead Time</td>
<td>1 period</td>
<td>Partial Shipping</td>
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<tr>
<td>Transportation Lead</td>
<td>1 period</td>
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<tr>
<td>Time</td>
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Case 2: Stochastic Demand

Case 2: Net Costs

Conclusions

- A control-oriented approach to supply chain management using MPC control with anticipation has been demonstrated.
- As an optimizer MPC is able to satisfy demand, while meeting constraints.
- As a controller MPC can be tuned for robustness and it is able to handle plant-model mismatch.

Future Work

- Non-linear Predictive Control via Model On-Demand Estimation
- Application to Multi-node, Multi-product Supply Chain Systems

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- NSF Grant DMI-0075682.