A Control-Relevant, Plant-friendly System Identification Methodology using Shifted and “Zippered” Multisine Input Signals

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Abstract

Informative input signals that are friendly to process operations are highly desirable in identification practice, with the goal of finding a control-relevant model estimate within an acceptable time-period. For shorter yet informative signals a “phase-shifted” design approach is applied to multisine input signal in this paper that reduce the overall identification test duration as an alternative plant-friendly multi-channel input design. This contrasts the previously developed approach using orthogonal (“zippered”) power spectra, which although extremely flexible, will lead to longer duration tests that may not be desirable for problems with large number of channels. Moreover, a control-relevant parameter estimation algorithm is developed in this paper for curvefitting Empirical Transfer Function Estimates (ETFEs) with orthogonal (i.e., zippered) frequency grids to discrete-time parametric Matrix Fraction Description models. Such ETFEs arise from DFT analysis of identification data generated from constrained, plant-friendly multisine inputs as developed by the authors’ previous work. This curvefitter minimizes model estimation error using pre/post frequency-dependent weighting matrices as functions of the closed-loop dynamics. The control-relevant multivariable parameter estimation procedure is illustrated with an example case study based on the Shell Heavy Oil Fractionator problem.

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1 Introduction

Informative input signals that are friendly to process operations are highly desirable in identification practice, with the goal of finding a meaningful model estimate within an acceptable time-period. Theoretical requirements in system identification encourage long test durations under conditions of high signal-to-noise ratios (Ljung, 1999), making system identification among the most expensive and time-consuming steps in the implementation of advanced control. Such identification testing conditions, however, may be hostile to the plant by causing additional “wear and tear” on process equipment while disrupting the normal process operations. The plant-friendliness in system identification takes consideration into a shorter test duration, minimum variations in outputs, and small move sizes and actuator limits in inputs. We have previously demonstrated practical usefulness of the zippered multisine signals as plant-friendly inputs for system identification in our recent work (Lee et al., 2003b). However, Kothare and Mandler (2003) point out that the zippered multisine inputs would be longer than necessary for large-scaled industrial processes in spite of their flexibility for multivariable systems; they propose that time-domain shifted Binary Multi-Frequency (BMF) signal inputs for a plant-friendly testing of a shorter duration.

In this paper, a “phase-shifted” design approach (Briggs and Godfrey, 1966; Rivera and Jun, 2000) is applied to a design of multisine input signals that reduce the overall identification test duration when the frequency independence is not required for multiple input channels. This contrasts the previously developed approach using orthogonal (“zippered”) power spectra, which although extremely flexible, will lead to longer duration tests that may not be desirable for problems with large number of channels (Lee et al., 2003b). Moreover, we will show an integrated identification procedure that uses a dataset, executed in a plant-friendly manner, for control-relevant parameter estimation problem (CRPEP) (Gaikwad and Rivera, 1996) via a frequency-weighted curvefitting algorithm based on the full-matrix polynomial structure such as Matrix Fraction Description (MFD) (de Callafon et al., 1996). The control-relevant, plant-friendly identification has received significant attention as informative and practical identification methodology that can be presented in a manner transparent to process engineers.

For multivariable system identification, shifting a single-channel signal in time-domain is a common technique to implement multi-channel input signals, e.g., PRBS, PRS, etc (Briggs and Godfrey, 1966; Godfrey, 1993; Rivera and Jun, 2000). The base and shifted signals share the same power spectrum density in the frequency-domain. On the contrary, multisine signals by the zippered power spectrum design possess orthogonality in each input channel that allows greater flexibility at the cost of extended sequence length. However, a length of the zippered power spectrum signal is increased by the number of input channels since multiple channels need wider frequency grids for orthogonality; whereas, shifting does not change the length since all channels share the same frequency grids.

Therefore, a time-domain shift method is applied in this paper to multisine input signals with a modifi-
cation from the guideline previously described for PRBS inputs in Rivera and Jun (2000). The determination of a shift parameter in the time-domain is implemented for multisine inputs in order to minimize the cross correlation between the input channels while their cross input power spectra are nonzero. For shifted multisine inputs we will discuss a brief guideline of PRBS input, its modification leading to shifted multisine input, and a comparison of PRBS and multisine inputs via a case study based on the Shell heavy-oil fractionator (Prett and García, 1988).

As an effective tool for model implementation in advanced control systems, control-relevant system identification has been studied extensively by control engineers, e.g., (Bayard, 1994; de Callafon et al., 1996; Gaikwad and Rivera, 1997). An integrated system identification and control methodology can provide a more appropriate model for control system design, possibly at the cost of compromising the open-loop fit (Gaikwad and Rivera, 1997). While traditional identification emphasizes open-loop model fits, the adequacy of a model for control design lies in its ability to generate a desired closed-loop response for a given set of set-points and disturbances. These distinctive goals often involve several back-and-forth iterations between identification and controller design procedures until an adequate model is obtained. Therefore, we consider control-relevancy in the parameter estimation step following identification testing, in order to produce a model estimate leading to a design that will meet closed-loop requirements. Specifically, a control-relevant parameter estimation algorithm is presented in this paper for curvefitting ETFEs with orthogonal (i.e., zippered) frequency grids to discrete-time Matrix Fraction Description (MFD) parametric models. Such ETFEs arise from the DFT analysis of identification data generated from constrained, plant-friendly multisine inputs as previously developed by Rivera et al. (Rivera et al., 2002) and H. Lee et al. (Lee et al., 2003b).

Previous work on a multivariable frequency response curvefitting includes an approach using a scalar polynomial as a common denominator with a scalar frequency-dependent weighting function proposed by Bayard (Bayard, 1994). Rivera and Gaikwad (Gaikwad and Rivera, 1997) extend Bayard’s approach (Bayard, 1994) so that a different denominator polynomial can be estimated for each output channel by the use of diagonal matrix polynomial. They also apply pre/post frequency-dependent weighting matrices to support control relevance. de Callafon et al. (de Callafon et al., 1996) apply a full-matrix polynomial MFD model that allows the flexibility to specify individual transfer function elements in multivariable systems. A Matrix Fraction Description (MFD) model representation is utilized in the curvefitting of frequency responses from the “zippered” multisine input signals to define a coherent, integrated identification testing and control-relevant parameter estimation strategy.

As suitable to multivariable control system designs we present a frequency-weighted curvefitting algorithm that utilizes a dataset executed in a plant-friendly way for a control-relevant parameter estimation procedure. The model representation based on full-matrix polynomials is implemented in our curvefitting algorithm and expanded to support zippered frequency grids with possible harmonic suppression that
arise from the use of multisine signals. A numerical approach based on Sanathanan-Koerner iteration (Sanathanan and Koerner, 1963) and Gauss-Newton optimization is utilized in the algorithm (Bayard, 1994). A modified system taken from a subset of the Shell fractionator plant (Prett and García, 1988) is used as an example study that illustrates how the model estimation error is shifted in frequency for achieving the control-relevance by the curvefitter. Ultimately, the goal of our work is to develop a plant-friendly system identification framework based on identification test monitoring that is able to produce a robust model for control design for large, nonlinear, multivariable process systems, such as those frequently encountered in advanced control implementations in the petrochemical and refining industries.

This paper is organized as follows: Section 2 describes the designs of zippered multisine, shifted PRBS, and shifted multisine input signals. Section 3 presents a comparison of open-loop identification test signals using shifted PRBS and shifted/zippered multisine input, based on a subset of the Shell heavy-oil fractionator plant. Section 4 focuses on a control-relevant curvefitting algorithm development and solves the numerical optimization problem for control-relevant parameters. Section 5 describes an example case study of the control-relevant parameter estimation and Section 7 presents summary and conclusions.

2 Designs of Zippered and Shifted Multisine Input Signals

Theoretical requirements for system identification is considered in frequency-domain while practical consideration such as plant-friendliness results in applying constrained optimization techniques to the input signal procedure (see Lee et al. (2003b)). The zippered multisine input signal and shifted PRBS signal designs are briefly discussed followed by a shifted multisine input signal design which provides a shorter signal length when frequency independence is not required for model estimation. Traditionally, time-domain shifted PRBS signals are used for multi-channel inputs in multivariable system identification; this time-domain shift technique is implemented in this paper to multisine inputs with minimal modification from the previously developed design guideline.

2.1 Design of Zippered Multisine Input Signals

Multisine signals are deterministic, periodic signals whose power spectrum can be directly specified by the user. A multisine input $u_j(k)$ for the $j$-th channel of a multivariable system with $m$ inputs can be defined as,

$$u_j(k) = \sum_{i=1}^{m\delta} \delta_{ji} \cos(\omega_i kT + \phi_{ji}) + \sum_{i=m\delta+1}^{m(\delta+n_j)} \alpha_{ji} \cos(\omega_i kT + \phi_{ji}) + \sum_{i=m(\delta+n_j)+1}^{m(\delta+n_j+n_a)} \tilde{a}_{ji} \cos(\omega_i kT + \phi_{ji}^a), \quad j = 1, \ldots, m$$

(1)
Figure 1: A conceptual design of a standard “zippered” spectrum for a three-channel signal.

where $T$ is sampling time, $N_s$ is the sequence length, $m$ is the number of channels, $\delta$, $n_s$, $n_a$ are the number of sinusoids per channel ($m(\delta + n_s + n_a) = N_s/2$), $\phi_{\delta ji}$, $\phi_{ji}$, $\phi_{a ji}$ are the phase angles, $\alpha_{ji}$ represents the Fourier coefficients defined by the user, $\hat{\delta}_{ji}, \hat{a}_{ji}$ are the “snow effect” Fourier coefficients (Guillaume et al., 1991), and $\omega_i = 2\pi i/N_s T$ is the frequency grid. To achieve a zippered spectrum we define the Fourier coefficients $\alpha_{ji}$ as:

$$\alpha_{ji} = \begin{cases} 
\neq 0, & i = m\delta + j, m(\delta + 1) + j, \ldots, m(\delta + n_s - 1) + j \\
= 0, & \text{for all other } i \text{ up to } m(\delta + n_s)
\end{cases} \quad (2)$$

Equivalent expressions to (2) can be developed for the “snow effect” coefficients $\hat{\delta}_{ji}$ and $\hat{a}_{ji}$.

In the design procedure, the primary frequency band of interest for excitation is determined by the dominant time constants of the system to be identified and the desired closed-loop speed-of-response,

$$\omega_s = \frac{1}{\beta_s \tau_{dom}^L} \leq \omega \leq \omega^* = \frac{\alpha_s}{\tau_{dom}^H} \quad (3)$$

$\alpha_s$ and $\beta_s$ are parameters that specify the high and low frequency ranges of interest in the signal, respectively for a given range of low and high dominant time constants (defined by $\tau_{dom}^L$ and $\tau_{dom}^H$). The primary band of excitation is bounded by the following inequality based on the choice of design parameters,

$$\frac{2\pi m(1+\delta)}{N_s T} \leq \omega_s \leq \omega \leq \frac{2\pi mn_s(1+\delta)}{N_s T} \leq \frac{\pi}{T} \quad (4)$$

which in turn translates into the following inequalities for sampling time, number of sinusoids, and se-
sequence length \( (T, n_s, \text{and } N_s, \text{respectively}) \):

\[
(1 + \delta) \frac{\omega^s}{\omega_s} \leq n_s \leq \frac{N_s}{2m} \tag{5}
\]

\[
T \leq \min \left( \frac{\pi}{\omega^s}, \frac{\pi}{\omega^s - \omega_s} \left(1 - \frac{1 + \delta}{n_s}\right) \right) \tag{6}
\]

\[
\max \left( 2mn_s, \frac{2\pi m(1 + \delta)}{\omega_s T} \right) \leq N_s \leq \frac{2\pi mn_s}{\omega^s T} \tag{7}
\]

As shown in Figure 1, the shape of the power spectrum in a multisine input is specified by the choice of Fourier coefficients. In addition a “notch” spectrum design can be used, with potentially variable number of Fourier coefficients in the low frequency area, primary frequency band, and high frequency area.

Theoretical requirements such as persistence of excitation, harmonic suppression (a key consideration in the identification of nonlinear systems), and control-relevance can be satisfied without loss of generality through the specification of Fourier coefficients (Rivera et al., 2002). Moreover, a modified zippered power spectrum has been recently designed, specially for highly interactive systems, that demonstrates the effectiveness of improving the gain directional contents in the dataset through the conventional open-loop identification tests (Lee et al., 2003b; Lee et al., 2003a). For more detailed information for the design guideline of plant-friendly multisine and constrained optimization techniques for time-domain constraints the readers can be referred to Lee et al.(2003a), Lee et al.(2003b), and Rivera et al.(2004).

### 2.2 Design of Shifted PRBS Inputs

The PRBS is a two-level, periodic, deterministic signal generated by using shift register modulo 2 addition. The PRBS sequence is characterized by two parameters, the number of registers \((n_r)\) and switching time or the clock period \((T_{sw})\), which is the minimum time between changes in the level of the signal. The signal repeats itself after \(N_s T_{sw}\) units of time, where \(N_s = 2^{n_r} - 1\). The persistent excitation bandwidth is obtained by applying a priori knowledge of a system like multisine input such as (3). The primary bandwidth (see (2)) serves as a basis for specifying design variables of PRBS. Following the analysis in Gaikwad and Rivera (1996), one can obtain expressions for specifying \(T_{sw}\) and \(n_r\) based on the parameters from (3)

\[
T_{sw} \leq \frac{2.8 \tau^H_{dom}}{\alpha_e} \tag{8}
\]

and a shift parameter for multiple channels is

\[
D \geq \frac{5 \tau^H_{dom}}{T_{sw}} \tag{9}
\]

The sequence length is then determined by

\[
N_s^{(1)} = \frac{2\pi \beta_s \tau^H_{dom}}{T_{sw}} \tag{10}
\]

\[
N_s^{(2)} = m \times D \tag{11}
\]
\[ N_s = 2^n_r - 1 \geq \max(N_s^{(1)}, N_s^{(2)}) \] (12)

where \( n_r \) and \( N_s \) must be integer values; \( T_{sw} \) and \( D \) must be integer multiples of the sampling time \( T \) and \( T_{sw} \), respectively.

Since simultaneous multiple input testing offers some advantages, among them the potential for decreasing the overall duration of experimental testing (Rivera and Jun, 2000). To generate a multi-input PRBS signal, the shift parameter (per 9) is utilized for keeping cross-correlation low enough between the channels. The identical number of registers and switching time are used in each channels; however, the registers must be initialized such that the input appears shifted relative to the previous input by a delay \( D \). Inputs numbered from 2 to \( m \) (the total number of channels) will be delayed by \([D, 2D, 3D, \ldots, (m - 1)D]\) with respect to the base input channel.

For an example with the Shell heavy-oil fractionator problem per (25), the signal design parameters for PRBS are found: \( \tau_{h_{dom}}^L = 74\text{min}, \tau_{l_{dom}}^L = 48\text{min}, \alpha_s = 2, \beta_s = 3 \). Accordingly, a primary bandwidth is defined as \( \omega^* = 0.04167 \) and \( \omega_0 = 0.0045 \). The choices for a PRBS input signal is that \( N_s = 496, T = 4\text{min} \) and \( T_{sw} = 64\text{min} \). We assume that there are three channels \( (m = 3) \) and a shift parameter is selected as \( D = 166 \times T (= 664\text{min}) \). The base PRBS signal and its shifted signals are shown in Figure 2.

Figure 2: Shell Heavy-Oil Fractionator Problem Example per 25: base and shifted PRBS signal for 3-channel inputs \( \tau_{h_{dom}}^L = 74\text{min}, \tau_{l_{dom}}^L = 48\text{min}, \alpha_s = 2, \beta_s = 3, N_s = 496, T = 4\text{min}, T_{sw} = 64\text{min} \), and \( D = 166 \times T (= 664\text{min}) \)
2.3 Design of Shifted Multisine Input Signal

We present an example that two-channel input signals are generated, sharing the same frequency grids: first, a base channel input is obtained by the general multisine equation and the second input is a shifted signal with respect to the base input. The first channel input is given as follows

\[ u_1(k) = \lambda \sum_{i=1}^{n_s} \sqrt{2} \alpha_i \cos(\omega_i k T + \phi_i) \]  

(13)

and this signal is shifted by a delay \( D \) for the second channel as the shifted PRBS signals

\[ u_2(k) = \lambda \sum_{i=1}^{n_s} \sqrt{2} \alpha_i \cos(\omega_i (k + D) T + \phi_i) \]  

(14)

where \( D \) is a shift parameter as defined in PRBS input guideline. In multisine input, \( D \) is added with the sequence index \( k \) for a delay in time-domain sequence. As a result, shifted multisine input signals are obtained by

\[ u_j(k) = \lambda \sum_{i=1}^{n_s} \sqrt{2} \alpha_i \cos(\omega_i (k + D_j) T + \phi_i) \]  

(15)

The shift parameter needs to be as large as possible to keep cross-correlation low enough for model estimation such as

\[ D_j = \frac{N_s}{m} \times (j - 1), \quad j = 2, ..., m \]  

(16)

The shifted channels, therefore, have a delay \( D \) as \( \frac{N_s}{m} \) which considers the settling time requirement (see PRBS shift parameter per (9)). \( D \) per (16) needs to satisfy the following inequality

\[ D = \frac{N_s}{m} \geq \frac{\beta_s \epsilon_{\text{dom}}^T}{T} = \frac{1}{\omega^* T} \]  

(17)

where \( \epsilon_{\text{dom}}^T \) in (9)) is replaced by \( \beta_s \) as a factor for the settling time of the process and \( T_{sw} \), by \( T \) as a sampling time for a multisine input signal. Then, \( N_s \) of a base channel should be

\[ N_s \geq \frac{m}{\omega^* T} \]  

(18)

which indicates that the base channel input length should be increased long enough for shifted inputs. As a result, we have the following inequality for \( N_s \)

\[ \max\{2n_1, \frac{2\pi(1 + \delta)}{\omega_s T}, \frac{m}{\omega_s T}\} \leq N_s \leq \frac{2\pi n_s}{\omega^* T} \]  

(19)

in which it must satisfy

\[ \frac{m}{\omega_s T} \leq N_s \leq \frac{2\pi n_s}{\omega^* T} \]  

(20)

namely, \( n_s \) should be

\[ n_s \geq \frac{m \cdot \omega^*}{2\pi \omega_s} \]  

(21)
The consideration of the shift parameter results in an inequality of \( n_s \).

For the use of shifted multisine signal in \( m \)-input channels, a modified guideline for a base input channel is presented as follows

\[
n_s \geq \max\{(1 + \delta) \frac{\omega'^*}{\omega}, \frac{m \omega'^*}{2\pi \omega_s}\}
\]  

\[
\max\{2n_s, \frac{2\pi(1 + \delta)}{\omega, T}\} \leq N_s \leq \frac{2\pi n_s}{\omega^* T}
\]

when \( \delta = 0 \) and \( m > 2\pi(\approx 7) \), \( n_s \) is determined by \( \frac{m \omega'^*}{2\pi \omega_s} \). Then \( N_s \) is given by

\[
N_s \leq \frac{m}{\omega_s} = \frac{\beta_s \tau_H^{H dom}}{T} = m \times D
\]

3 Comparison of Shifted PRBS and Shifted/Zippered Multisine Signals

For multi-channel inputs we have discussed how PRBS and multisine signals are generated to achieve plant-friendly yet informative identification tests. Now users can have a choice among shifted PRBS, shifted multisine, and zippered multisine input signals for multivariable systems. An example case study based on the Shell heavy-oil fractionator process that is presented for a comparison of the use of these three signal designs.

3.1 Shell Heavy-Oil Fractionator

For the Shell Heavy-Oil Fractionator problem, the signal design parameters are obtained from \textit{a priori} knowledge of the system: \( \tau_H^{H dom} = 74 \) and \( \tau_L^{H dom} = 48 \) where the Shell Heavy-Oil Fractionator is given as

\[
y(t) = \begin{bmatrix}
\frac{1.77}{60s+1} \exp^{-28s} & \frac{5.88}{60s+1} \exp^{-27s} \\
\frac{5.72}{60s+1} \exp^{-14s} & \frac{6.90}{40s+1} \exp^{-15s}
\end{bmatrix} u(t)
\]

where the sampling time, \( T = 4 \) min, the physical outputs and inputs are

\[
y(t) = \begin{bmatrix}
\text{Top End. Product} \\
\text{Side End. Product}
\end{bmatrix} \quad u(t) = \begin{bmatrix}
\text{Side Draw} \\
\text{BR Duty}
\end{bmatrix}
\]

3.2 Open-Loop Identification Test

A series of PRBS and multisine signals are generated using the same information, the design parameters of signals are listed in Table 1. All the cases meet the frequency bandwidth requirements of \( \omega_s \) and \( \omega'^* \). Interestingly, the shifted multisine signal has a shorter signal length than the PRBS case by 29.43%. As
expected, the zippered multisine signal is the longest case which is two times longer than the shifted multisine signal.

<table>
<thead>
<tr>
<th>Signal</th>
<th>$\alpha_s$</th>
<th>$\beta_s$</th>
<th>$\omega_s$</th>
<th>$\omega^*$</th>
<th>$T$ (min)</th>
<th>$N_s$</th>
<th>$1_{cycle}$ (min)</th>
<th>$D$</th>
<th>CF($x$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRBS (shifted)</td>
<td>2</td>
<td>3</td>
<td>0.0045</td>
<td>0.0417</td>
<td>4</td>
<td>496</td>
<td>1984.00</td>
<td>248</td>
<td>1.000</td>
</tr>
<tr>
<td>Multisine (shifted)</td>
<td>2</td>
<td>3</td>
<td>0.0045</td>
<td>0.0417</td>
<td>4</td>
<td>350</td>
<td>1400.00</td>
<td>175</td>
<td>1.421</td>
</tr>
<tr>
<td>Multisine (zippered)</td>
<td>2</td>
<td>3</td>
<td>0.0045</td>
<td>0.0417</td>
<td>4</td>
<td>698</td>
<td>2792.00</td>
<td>0</td>
<td>1.218</td>
</tr>
</tbody>
</table>

Table 1: Shell Heavy-Oil Fractionator Problem per 25 : signal design parameters (for PRBS, $T_{sw} = 64$ min; multisine signals are generated using Guillaume-phasing, $hf = 0$ for the shifted and zippered case) and crest factor (CF) is defined as $CF(x) = \frac{\ell_{\infty}(x)}{\ell_2(x)}$

The comparison of input power spectra, time series signal, and correlation analysis is given in the following figures. First, the comparison of input power spectra is shown in Figure 3. The shifted signals power spectra show that they are correlated in the sense of frequency-domain; they might have poor identifiability using frequency responses for simultaneous multivariable input signal usage. Figure 4 shows the time-domain sequences of input and output signals of PRBS and multisine signals. The shifted PRBS and multisine signals have relatively shorter lengths than the zippered multisine signals.

The time-domain correlation analysis is shown in Figure 5, and variance and standard error bounds are given as follows

$$Var[\gamma_{xx}(k)] \approx \pm \frac{1}{N}, \quad k > 0$$  \hspace{1cm} (27)

and the standard error bound is given by

$$S.E[\gamma_{xx}(k)] \approx \pm \frac{3}{\sqrt{N}}, \quad k > 0 \quad \text{with 99% confidence}$$  \hspace{1cm} (28)

For cross-correlation analysis, the variance bound is

$$Var[\gamma_{xe}(k)] \approx \pm \frac{3}{N-k}, \quad k > 0$$  \hspace{1cm} (29)

The cross-correlation of the shifted signals do not violate the variance criteria with a suitable shift parameter; specially, this means that the shifted multisine signal can have low cross-correlation between the base input channel. When a lag of the correlation analysis is close to $D$ (shift parameter), the correlation is increasing values over the confidence level. Therefore, $D$ should be large enough in the input design procedure based on the settling time of a process to reduce the cross correlation effect in model estimation. On the other hand, the zippered multisine signal will not violate this variance criteria in the correlation since their inputs are independent.

\hspace{1cm} (de Callafon et al., 1996).
Figure 3: Shell Heavy-Oil Fractionator Problem Example per (25): Input power spectra of PRBS (shifted, (a)), Multisine (shifted, (b)), and Multisine (zippered, (c)) signals
Figure 4: Shell Heavy-Oil Fractionator Problem Example per (25): Time sequences of input and output signals of PRBS (shifted, (a)), Multisine (shifted, (b)), and Multisine (zippered, (c)) signals.
Figure 5: Shell Heavy-Oil Fractionator Problem Example per (25): Correlation analysis of input signals with lag=15, PRBS (shifted, (a)), Multisine (shifted, (b)), and Multisine (zippered, (c)) signals.
4 Control-Relevant Curvefitting for Plant-Friendly System Identification

4.1 Control-Relevant Parameter Estimation

A key feature of control-relevant parameter estimation is that emphasizes closed-loop requirements during the estimation procedure. In other words, the goal is to obtain a model \( \tilde{P} \) representing a system \( P \) that is best suited for the end use of model, which is control system design. To this end, the work of Gaikwad and Rivera (Gaikwad and Rivera, 1997) established that such a parameter estimation problem can be cast as a pre- and post-weighted 2-norm minimization,

\[
\min_E \| G_{21}E G_{12} \|_F^2 \approx \min_E \| W_E \tilde{S} E_{m} \tilde{H} (r - d) \|_2^2
\]

subject to the condition that

\[
\sup_\omega \rho (E_{m} \tilde{H}) < 1, \quad -\pi \leq \omega \leq \pi
\]

where the pre- and post-weights are functions of the closed-loop transfer functions as \( \tilde{S} = (I + \tilde{PC})^{-1} \), \( \tilde{H} = \tilde{PC}(I + \tilde{PC})^{-1} \) and \( E_{m} = (P - \tilde{P}) \tilde{P}^{-1} \). \( \rho (E_{m} \tilde{H}) \) arises from the Small Gain Theorem and can be used as a sufficient condition for nominal stability. Control-relevant weights defined by \( G_{12} \) and \( G_{21} \) will dictate which frequency bandwidth is the most important for control system design. For a more detailed discussion of the CRPEP formulation the reader can be referred to (Gaikwad and Rivera, 1997).

4.2 Matrix Fraction Description Model Representation

A consistent estimate of the frequency response is obtained from the observed input and output data via an ETFE. The frequency responses are averaged from \( r \)-cycles of input and output data by DFT analysis according to

\[
g_{h\ell}(\omega_k) = \frac{1}{r} \sum_{s=0}^{r-1} Y_h^s(\omega_k) \]

\[
Y(\omega_k) = DFT(y), \quad U(\omega_k) = DFT(u)
\]

where \( g_{h\ell}(\omega_k) \) is computed on frequencies when \( U(\omega_k) \neq 0 \).

Extended to multivariable systems a frequency-domain identification procedure using the discrete-time MFD is described in (de Callafon et al., 1996) where frequency responses are given as

\[
\mathcal{G} := \{ G(\omega_k) | G(\omega_k) \in \mathbb{C}^{p \times m}, \text{ for } i \in [1,2,...,N] \}
\]

The frequency response data set \( \mathcal{G} \) consists of complex matrices \( G(\omega_k) \). A model \( \tilde{P} \) of \( m \) inputs and \( p \) outputs is approximated into a linear parametric, real rational transfer function, formed by either a left or
right matrix polynomial fractional description.

\[
\text{Left MFD } \hat{P}(\xi^{-1}, \theta) = A(\xi^{-1}, \theta)^{-1}B(\xi^{-1}, \theta) \tag{34}
\]

\[
\text{Right MFD } \hat{P}(\xi^{-1}, \theta) = B(\xi^{-1}, \theta)A(\xi^{-1}, \theta)^{-1} \tag{35}
\]

where \( \xi(\omega_t) = j\omega_t \) in a continuous time model, whereas \( \xi(\omega_t) = e^{j\omega T} \) represents the shift operator in a discrete-time model. For both the left and right MFD, the polynomial matrix \( B \) is defined as a function of \( \xi^{-1} \) and \( \theta \)

\[
B(\xi^{-1}, \theta) = \sum_{k=d}^{d+b-1} B_k \xi^{-k}, \ B_k \in \mathbb{R}^{p \times m} \tag{36}
\]

where \( d \) denotes the number of leading zero matrix coefficients and \( b \) the number of non-zero matrix coefficients in the \( B(\xi^{-1}, \theta) \). For the left MFD, the \( A \) polynomial is parameterized by

\[
A(\xi^{-1}, \theta) = I_{p \times p} + \xi \sum_{k=1}^{a} A_k \xi^{-k+1}, A_k \in \mathbb{R}^{p \times p} \tag{37}
\]

and for the right MFD

\[
A(\xi^{-1}, \theta) = I_{m \times m} + \xi \sum_{k=1}^{a} A_k \xi^{-k+1}, A_k \in \mathbb{R}^{m \times m} \tag{38}
\]

where \( a \) denotes the number of non-zero coefficients in the polynomial \( A(\xi^{-1}, \theta) \). The parameter \( \theta \) consists of the corresponding unknown matrix coefficients in the \( A \) and \( B \) polynomials. In this paper, we utilize the left MFD parameterization for control-relevant parameter estimation purposes.

The structural parameters \( d_{i,j}, b_{i,j} \) and \( a_{i,j} \) that are specified for each of the elements of the polynomial matrices \( A \) and \( B \) separately, with

\[
d := \min\{d_{i,j}\}, \ b := \max\{b_{i,j}\}, \ a := \max\{a_{i,j}\} \tag{39}
\]

The model order of polynomial estimation via MFD has a limitation on the McMillan degree of the resulting estimate \( \hat{P}(\xi, \theta) \) instead of asymptotic higher orders in ARX models. For a more detailed discussion on the exact relation between the McMillan degree, the row degree of the polynomial matrices \( A(\xi^{-1}, \theta), B(\xi^{-1}, \theta) \) and the observability indices of a model computed by \( A(\xi^{-1}, \theta)^{-1}B(\xi^{-1}, \theta) \) the reader can be referred to (Gevers, 1986; Van den Hof, 1992).

The model error between the process and model is represented as

\[
E(\omega, \theta) = G(\omega_t) - \hat{P}(\omega_t) \text{ for } i \in [1,2,...,N] \tag{40}
\]

where \( \hat{P} \) is given by Left-MFD (34) and \( E(\omega, \theta) \) is

\[
E(\omega, \theta) = G(\omega_t) - A(\xi(\omega_t)^{-1}, \theta)^{-1}B(\xi(\omega_t)^{-1}, \theta) = A(\xi(\omega_t)^{-1}, \theta)^{-1}\hat{E}(\omega_t) \tag{41}
\]
where \( \tilde{E}(\omega_i) = G(\omega_i) - \theta \Phi(\omega_i) \) and \( \theta \) and \( \Phi \) are given as

\[
\theta = [B_d \ldots B_{d+b-1} A_1 \ldots A_a] \in \mathbb{R}^{n \times (mb+pa)}
\]

(42)

\[
\Phi(\omega_i) = \begin{pmatrix}
I_{m \times m} \xi(\omega_i)^{-d} \\
\vdots \\
I_{m \times m} \xi(\omega_i)^{-(d+b-1)} \\
G(\omega_i) \xi(\omega_i)^{-1} \\
\vdots \\
G(\omega_i) \xi(\omega_i)^{-a}
\end{pmatrix}
\]

(43)

If zippered multisine signals are applied for simultaneously exciting all the input channels, a permutation matrix, \( T_m \), should be applied to ensure only the relevant frequencies in the MFD models are evaluated such that

\[
\tilde{E}(\omega_i) = (G(\omega_i) - \theta \Phi(\omega_i)) T_m(\omega_i)
\]

(44)

where \( T_m \) is defined by

\[
T_m(\omega_i) = \text{diag}(0, \ldots, 1_{j_{th}}, \ldots, 0), \quad T_m \in \mathbb{R}^{m \times m}
\]

(45)

If \( j_{th} \) input channel has non-zero power at \( \omega_i \) based on the zippered frequency grids, only \( T_m^{(j_{th})}(\omega_i) = 1 \) and all the other elements are zero. \( T_m(\omega_i) \) can also be applied to harmonically-suppressed frequency grids, which is a consideration in the identification of nonlinear systems.

### 4.3 Frequency-Weighted Curve-fitting

Non-weighted parameter estimation can be directly accomplished from the minimization of \( \| E(\omega_i, \theta) \|_2^2 \), which is solved by an iterative least squares method, i.e., Sanathanan-Koerner (SK) iteration method (Sanathanan and Koerner, 1963). The S-K model provides an initial set of parameters for Gauss-Newton optimization (Bayard, 1994).

Incorporating with control-relevant weights, the weighted error is represented as

\[
\tilde{E}_w(\omega_i, \theta) = \tilde{W}_2(\omega_i, \theta) \tilde{E}(\omega_i, \theta) W_1(\omega_i, \theta)
\]

(46)

where \( \tilde{W}_2(\omega_i, \theta) = -W_2(\omega_i) \tilde{S}(\omega_i, \theta) A(\omega_i, \theta)^{-1} \) and \( W_1(\omega_i, \theta) = \tilde{P}^{-1}(\omega_i, \theta) H(\omega_i, \theta)(r-d) \). All the matrices in \( \tilde{E}_w \) are dependent on frequency and parameters. Thus, an iterative procedure is utilized in this problem such as (Sanathanan and Koerner, 1963). The \( \theta \) in step \( t \) is obtained by taking the weights, \( W_2(\omega_i, \theta_{t-1}) \) and \( W_1(\omega_i, \theta_{t-1}) \), based on the previous model parameter \( \theta_{t-1} \). This iterative weighted error
equation is denoted by

\[ \tilde{E}_w(\omega_i, \theta_{t-1}, \theta) = \tilde{W}_2(\omega_i, \theta_{t-1}) \tilde{E}(\omega_i, \theta) W_1(\omega_i, \theta_{t-1}) \]

\[ = \tilde{W}_2(\omega_i, \theta_{t-1}) [G(\omega_i) - \theta \Phi(\omega_i)] T_m(\omega_i) W_1(\omega_i, \theta_{t-1}) \Delta \omega_i \] (47)

Now a parameter vector \( \theta_t \) is estimated from the minimization of \( \tilde{E}_w(\omega_i, \theta_{t-1}, \theta) \) from

\[ \theta_t = \arg \min_{\theta \in \mathbb{R}^N} \sum_{k=1}^N \| \tilde{W}_2(\omega_k, \theta_{t-1}) \tilde{E}(\omega_k, \theta) W_1(\omega_k, \theta_{t-1}) \|^2 \Delta \omega_i \] (48)

where \( \Delta \omega_i \) represents the frequency interval for zippered or/and harmonic-suppressed input power spectra. However, the computation of minimization of (48) is much more difficult than a non-weighted error formulation because of the pre/post weights on \( \tilde{E}_w \). The Kronecker vector and Kronecker product are applied to the weighted error matrix.

With matrices \( X, Y, \) and \( Z \) with appropriate dimensions, the matrix product \( C = XYZ \) can be obtained by using Kronecker vector and Kronecker product such that

\[ \text{vec}(C) = [Z^T \otimes X] \text{vec}(Y) \]

Considering matrices \( X \in C^{m_1 \times n_2} \) and \( Y \in C^{m_1 \times m_2} \), the Kronecker vector \( \text{vec}(X) \) and Kronecker product \( X \otimes Y \) operators are defined by

\[ \text{vec}(X) := \begin{bmatrix} x_{1,1} \\ x_{2,1} \\ \vdots \\ x_{n_1,n_2} \end{bmatrix} \quad \quad \otimes \quad \begin{bmatrix} x_{1,1}Y & \ldots & x_{1,n_2}Y \\ \vdots & \ldots & \vdots \\ x_{1,1}Y & \ldots & x_{1,n_2}Y \end{bmatrix} \] (49)

where \( \text{vec}(X) \in C^{n_1 n_2 \times 1} \) and \( X \otimes Y \in C^{n_1 n_2 \times n_1 n_2} \).

As the Frobenius-norm is still valid with the Kronecker operator \( \| X \|^2_F = \| \text{vec}(X) \|^2_F \) for an arbitrary matrix \( X \), the minimization objective function of \( \tilde{E}_w \) can be rewritten in terms of the Kronecker operators. Now, \( \tilde{E}_w \) is rewritten as

\[ \tilde{E}_w = \tilde{W}_2 (G - \theta \Phi) T_m W_1 \]

\[ = (\tilde{W}_2 G T_m W_1) - (\tilde{W}_2 \theta \Phi T_m W_1) \] (50)

and taking Kronecker \( \text{vec} \) operator to \( \tilde{E}_w \)

\[ \text{vec}(\tilde{E}_w) = \text{vec}(\tilde{W}_2 G T_m W_1) - [(\Phi T_m W_1)^T \otimes \tilde{W}_2] \tilde{\theta} \]

\[ = G_{\omega} - \Phi_{\omega} \tilde{\theta} \] (51)
where \( vec(\theta) = \bar{\theta} \). As a result, a control-relevant parameter is obtained by

\[
\theta^C_{REP} = \arg \min_{\theta \in \mathbb{R}} \| \text{vec}(\vec{E}_w(\theta_{t-1}, \theta)) \|^2_T = \arg \min_{\theta \in \mathbb{R}} \| G_\omega - \Phi_\omega \bar{\theta} \|^2_T
\]

where \( G_\omega \) and \( \Phi_\omega \) are formulated as

\[
G_\omega := \begin{bmatrix}
\text{vec}(\mathcal{R}\{W_2(\omega_1, \theta_{t-1})G(\omega_1)T_m(\omega_1)W_1(\omega_1, \theta_{t-1})\Delta \omega_1\}) \\
\vdots \\
\text{vec}(\mathcal{R}\{W_2(\omega_N, \theta_{t-1})G(\omega_N)T_m(\omega_N)W_1(\omega_N, \theta_{t-1})\Delta \omega_N\}) \\
\text{vec}(\mathcal{Z}\{W_2(\omega_1, \theta_{t-1})G(\omega_1)T_m(\omega_1)W_1(\omega_1, \theta_{t-1})\Delta \omega_1\}) \\
\vdots \\
\text{vec}(\mathcal{Z}\{W_2(\omega_N, \theta_{t-1})G(\omega_N)T_m(\omega_N)W_1(\omega_N, \theta_{t-1})\Delta \omega_N\})
\end{bmatrix}
\]

\[
\Phi_\omega := \begin{bmatrix}
\mathcal{R}\{[\Phi(\omega_1)T_m(\omega_1)W_1(\omega_1, \theta_{t-1})\Delta \omega_1]^T \otimes \bar{W}_2(\omega_1, \theta_{t-1})\} \\
\vdots \\
\mathcal{R}\{[\Phi(\omega_N)T_m(\omega_N)W_1(\omega_N, \theta_{t-1})\Delta \omega_N]^T \otimes \bar{W}_2(\omega_N, \theta_{t-1})\} \\
\mathcal{Z}\{[\Phi(\omega_1)T_m(\omega_1)W_1(\omega_1, \theta_{t-1})\Delta \omega_1]^T \otimes \bar{W}_2(\omega_1, \theta_{t-1})\} \\
\vdots \\
\mathcal{Z}\{[\Phi(\omega_N)T_m(\omega_N)W_1(\omega_N, \theta_{t-1})\Delta \omega_N]^T \otimes \bar{W}_2(\omega_N, \theta_{t-1})\}
\end{bmatrix}
\]

The Kronecker operators transform the estimation of large-size models into a convenient matrix structure for seeking a solution of \( \theta \).

Taking the weighted error function into the one-step least squares method, the parameter \( \bar{\theta} \) is obtained as the minimum of the following function

\[
F = \| \text{vec}(\vec{E}_w) \|^2_T = (G_\omega - \Phi_\omega \bar{\theta})^T (G_\omega - \Phi_\omega \bar{\theta})
\]

and a solution is found when \( \frac{\partial F}{\partial \bar{\theta}} = 0 \)

\[
\bar{\theta} = (\Phi_\omega^T \Phi_\omega)^{-1} (\Phi_\omega^T G_\omega)
\]

As an iterative minimization procedure, the Hessian-Newton matrix is applied in this problem since \( F \) is twice differentiable with respect to \( \bar{\theta} \) and the iterative procedure is given

\[
\hat{\theta}_{k+1} = \hat{\theta}_k - [H(\theta_k)]^{-1} \nabla F(\hat{\theta}_k) = \hat{\theta}_k - [2\Phi_\omega^T \Phi_\omega]^{-1} [-2\Phi_\omega^T (G_\omega - \Phi_\omega \hat{\theta}_k)]
\]

where \( H(\theta_k) = 2\Phi_\omega^T \Phi_\omega \). For numerical convergence in the iteration, a set of termination criteria can be given as follows (see (Rivera and Morari, 1987))

\[
\left| \frac{\hat{\theta}_{k+1} - \hat{\theta}_k}{\hat{\theta}_{k+1}} \right| \leq \varepsilon_1
\]

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If the two criteria are satisfied simultaneously, we can terminate the iteration loop at step \( k + 1 \) with user specifications of \( \varepsilon_1 \) and \( \varepsilon_2 \). A flowchart of this control-relevant parameter estimation is illustrated in Figure 6.

5 Example of Control-relevant Parameter Estimation to Shell Heavy-Oil Fractionator

5.1 Problem Description and Experiment Execution

A \( 2 \times 2 \) multivariable system consisting of a subset of the Shell Heavy-Oil Fractionator Plant (Prett and García, 1988) is taken as an example case study for the control-relevant parameter estimation with MPC. The process is a linear and has time-delay in each transfer function element. A modified model from the Shell Heavy-Oil Fractionator is obtained by increasing the time-delay as

\[
\begin{align*}
    y(t) &= \begin{bmatrix}
    4.05 \exp^{-50s} & 1.77 \exp^{-60s} \\
    5.72 \exp^{-50s} & 5.72 \exp^{-60s}
    \end{bmatrix}
    \begin{bmatrix}
    u(t) + d(t)
    \end{bmatrix}
    \begin{bmatrix}
    1.44 \exp^{-27s} \\
    1.83 \exp^{-15s}
    \end{bmatrix}
\end{align*}
\]

where the physical outputs and inputs are

\[
\begin{bmatrix}
    y(t) = [\text{Top End. Product}; \text{Side End. Product}] \\
    u(t) = [\text{Top Draw}; \text{Side Draw}] \\
    d(t) = [\text{URDuty}]
\end{bmatrix}
\]

In this way, the increase in time delay creates the opportunity for large bias in model estimation which will contrast the weighted vs. unweighted approaches.

The input signal design parameters are obtained from a priori knowledge of the system: \( \tau_{\text{dom}}^H = 74 \), \( \tau_{\text{dom}}^L = 48 \), and feasible multisine design variables are obtained as \( T = 4\text{min} \), \( n_s = 10 \), \( N_s = 698 \), and \( h_f = 0.0 \). An open-loop identification experiment is performed using zippered multisine input signals(see Figure 7). This dataset is used for calculating ETFE values (Figure 8) and solving a control-relevant parameter estimation problem.

5.2 Solving control-relevant parameter estimation problem

We utilize ten cycles of data Under noisy conditions \( \sigma_d^2 = 2.0 \) to obtain frequency responses. These are shown in Figure 7. Tuning parameters for the MPC controller that defines the control-relevant weights are: \( \text{PH}=50 \), \( \text{MH}=10 \), \( \text{Ywt}=[1 1] \), \( \text{Uwt}=[32 40] \) with setpoint direction \([1 -1]\). The curvefits, shown in Figure 8, suffer mismatch in most of high frequencies while only the weighted curvefits are close to ETFEs in the
Supply frequency responses: $P(j\omega)$

Specify model orders $(n_A, n_B), \varepsilon_{Tol}$, max no. of iterations

Set initial weights $W_1^0, W_2^0$ from SK-iteration

Solve Least-Squares & G-N Minimization for CRPEP

$$E_v(\omega_i, \theta_t, \theta) = \tilde{W}_2(\omega_i, \theta_t) \tilde{E}(\omega_i, \theta) \tilde{W}_1(\omega_i, \theta_t) \Delta \omega_i$$

$$= \tilde{W}_2(\omega_i, \theta_t) [P(\omega_i) - \theta \Phi(\omega_i)] T_m(\omega_i) \tilde{W}_1(\omega_i, \theta_t) \Delta \omega_i$$

$$\theta_{t+1} = \arg \min_{\theta \in \mathbb{R}} \sum_{k=1}^{N} \| \tilde{W}_2(\omega_i, \theta_t) \tilde{E}(\omega_i, \theta) \tilde{W}_1(\omega_i, \theta_t) \|_2^2 \Delta \omega_i$$

Are

$$\left| \frac{\theta_{t+1} - \theta_t}{\theta_{t+1}} \right| \leq \varepsilon_1$$

$$\left| \frac{\| W_2 EW_1 \|_2^2 + \| W_2 EW_1 \|_2^2_{t+1}}{\| W_2 EW_1 \|_2^2_{t+1}} \right| \leq \varepsilon_2$$

satisfied?

Update weights from new model

$$\theta_t \leftarrow \theta_{t+1}$$

$$\tilde{W}_2(\omega_i, \theta_t) = -W_2(\omega_i) \tilde{S}(\omega_i, \theta_{t+1}) A^{-1}(\omega_i, \theta_{t+1})$$

$$\tilde{W}_1(\omega_i, \theta_t) = \tilde{P}^{-1}(\omega_i, \theta_{t+1}) \tilde{H}(\omega_i, \theta_{t+1})(r - d)$$

Control-relevant Model Parameter $	heta_{t+1}$

Figure 6: Flowchart for Control-relevant Parameter Estimation Algorithm
low frequencies. Figure 9 displays $\rho(E_m\tilde{H})$ for both the unweighted and weighted models. The spectral radius is shifted by applying the control-relevant weights such that the weighted model has lower values in the low frequencies but has higher values in the high frequencies than those of the unweighted case. Particularly, both models display nominal stability since they satisfy $\rho(E_m\tilde{H}) < 1$ condition. A comparison of open-loop step responses displays the difference in gain values between the unweighted and weighted MFD models and the plant (Figure 10). In closed-loop MPC setpoint tracking test, the weighted model has no offset in $y_1$ and $y_2$ while the unweighted suffers offset in $y_1$ and more oscillation in $y_2$ (Figure 11).

6 Summary and Conclusions

Based on a priori knowledge of a system, feasible design parameters are obtained for shifted PRBS and multisine inputs. The shifted signals of PRBS inputs are commonly used for multiple channels that is applied for multisine inputs with a necessary modification in its design guideline. Through the example case study, a time-domain shifting technique is easily implemented in the multisine input signal like PRBS for multiple input channels. Multisine signals for multivariable system identification, therefore, can be used as either shifted or zippered signals depending on the system characteristics.

A method for parametric model estimation from frequency-weighted curvefitting is achieved by the use of the multisine input signals and the full-matrix polynomial MFD approach. This MFD is implemented to
Figure 8: Frequency response curvefitting with MFD\([n_a = 1, n_b = 1, n_d = 1]\) models per (59) under noisy conditions, \(\sigma_d^2 = 2.0\) (solid: true plant, *: ETFE values, dashed: unweighted MFD, dotted: weighted MFD)

support the plant-friendly multisine inputs based on zippered frequency grids with possible harmonic suppression. The objective function of the CRPEP is numerically solved by iterative minimization procedures using S-K iteration and GN-optimization.

In the example case study, the weighted curvefitting shows the frequency-dependent error minimization utilizing the control-relevant pre/post-weights as functions of the closed-loop dynamics. Since the weighting functions emphasize the low-frequency dynamics in the example, the parameter estimation error is shifted into the high frequency area that is less relevant to the closed-loop control performance. Therefore, the control-relevant weights properly reflect the closed-loop dynamics in the curvefitting of frequency responses into linear MFD models. Future research considers integrating this work into a comprehensive identification test monitoring procedure.

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Figure 9: $\rho(E_n \tilde{H})$ comparison between the unweighted and weighted MFD[$n_a = 1, n_b = 1, n_d = 1$] models per (59)

Figure 10: Open-loop step responses of MFD[$n_a = 1, n_b = 1, n_d = 1$] models per (59)(solid: true plant, dashed: unweighted MFD, dotted: weighted MFD)
Figure 11: Closed-loop MPC setpoint tracking test using MFD\([n_a = 1, n_b = 1, n_d = 1]\) models per (59): PH=50, MH=10, Ywt=[1 1], & Uwt=[32 40] (solid: true plant, dashed: unweighted MFD, dash-dotted: weighted MFD, dotted: setpoint)

References


