DESIGN OF MINIMUM CREST FACTOR
MULTISINUSOIDAL SIGNALS FOR
“PLANT-FRIENDLY” IDENTIFICATION OF
NONLINEAR PROCESS SYSTEMS

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Abstract: Guidelines for specifying design parameters for minimum crest factor
multisine signals per the approach of Guillaume et al. are presented. These guidelines
are evaluated for the identification of nonlinear process systems. The minimum crest
factor multisine signals offer some distinct advantages over both Schroeder phased
multisine signals and m-level Pseudo-Random Sequence (m-level PRS) signals with
respect to “plant-friendliness” considerations. These signals can be used to reduce the
effects of nonlinearity in obtaining an empirical transfer function estimate (ETFE).
As an example, the ETFE of a Rapid Thermal Processing (RTP) reactor simulation
is constructed. “Plant-friendly” issues are also discussed and illustrated in the
identification and control of a CSTR simulation via “Model-on-Demand” estimation.
This provides a compelling example, since the “Model-on-Demand” estimator is a
data-driven nonlinear identification approach.

Keywords: nonlinear model identification, local modeling, nonlinear model
predictive control, multisine signals, input signal design

1. INTRODUCTION

Much effort has been devoted in recent years to
development of nonlinear identification of process
systems. Yet input signal design methods that
take into account practical considerations remain
an open area of research (Pearson and Ogun-
naiké, 1997). This work focuses on comparing m-
level Pseudo-Random Sequences (m-level PRS),
for which guidelines for identification of nonlinear
plants have been developed and applied (Braun
et al., 1999), against minimum crest factor multi-
sine signals proposed by Guillaume and coworkers
(Guillaume et al., 1991). These multisine signals
are designed based on the guidelines developed for
Schroeder-phased multisine signals for the SISO
case (Rivera et al., 1993). The minimum crest
factor multisine signals are shown to provide signifi-
cantly lower crest factor than Schroeder phased
signals, while having shorter signal length and
smaller maximum move size than m-level PRS
signals designed for the same bandwidth. The ben-
efits of harmonic suppression in identifying linear
dynamics are demonstrated on a Rapid Thermal
Processing (RTP) reactor simulation. Lastly, is-
issues regarding regressor space support are shown
to effect “Model-on-Demand” nonlinear identifi-
cation and control of a CSTR simulation. “Model-
on-Demand” provides a compelling method of
evaluation of these signals, since it is a data-driven
technique.

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2. MULTISINE SIGNAL DESIGN

GUIDELINES

m-level PRS signals are conveniently generated by a deterministic algorithm. However, a disadvantage of the m-level PRS signals is that desired frequency limits may not exactly be met by the set of sequences available. This results in a signal that is somewhat longer than is necessary or the user might have to overdesign the high frequency content to try to reduce the length of the signal. In the case that the input amplitude is severely restricted, the m-level PRS signal may not provide significant variance in the output compared to other classes of signals with lower crest factors. m-level PRS signals will switch in one switching time between the limits of the input range. Because of actuator move-size constraints and other plant limitations, this is not a “plant-friendly” property.

An attractive alternative is the use of a multisine signal because a desired spectrum can be met exactly. Multisine signals, like the m-level PRS, are deterministic, periodic signals. Multisine signals can be optimized to have a minimum crest factor and have no restrictions on the number of levels they can achieve within the input range. This provides better resolution of the input space which is desirable for nonlinear process identification. Prior experience with the time domain realization process is initialized with the phases proposed by the Schroeder-phase algorithm (Schroeder, 1970). In general this is a nonlinear optimization problem. Although a global solution cannot be guaranteed with this approach, most local minima are avoided and experimentally it performs better than other crest factor minimization techniques (Guillaume et al., 1991).

The guidelines of (Rivera et al., 1993) can be directly applied to ensure the signal contains the frequency information necessary to meet control relevant requirements

\[ \omega_s = \frac{1}{\beta \tau_{dom}^H} \leq \omega \leq \frac{\alpha}{\tau_{dom}^L} = \omega^* \]  

To meet these requirements \( N_s \) and \( n_s \) can be chosen such that

\[ \omega_s \geq \frac{2\pi}{N_s T} \quad \omega^* \leq \frac{2\pi n_s}{N_s T} \]  

or equivalently,

\[ N_s \geq \frac{2\beta \pi T}{\tau_{dom}^H} \quad n_s \geq \frac{N_s T \alpha}{2\pi \tau_{dom}^L}. \]

\( \tau_{dom}^H \) and \( \tau_{dom}^L \) correspond to the high and low estimates of the dominant time constant of the system. \( \alpha \) and \( \beta \) correspond to user-decisions on the high and low frequency content based on control requirements. The user also has the additional freedom to choose relative magnitudes of the harmonics \( \alpha \) for harmonic suppression or for any shape of a desired power spectrum. Harmonic suppression allows the user to decompose the output to obtain a more accurate estimate of the linear dynamics. Additional details regarding harmonic suppression can be found in (Godfrey, 1993). This work chooses to maintain a ’flat’ frequency dependence within the control relevant frequency range, with the option of additional high frequency content, as shown in Figure 1.

3. RTP CASE STUDY

The goal of this section is to show the improvement in crest factor gained by the Guillaume et al. algorithm over the Schroeder phased multisine and to demonstrate the benefit of harmonic suppression in calculating the ETFE. The noise-free RTP model originally posed by (Christoffel et al., 1995) is used as the truth model. The RTP approximating the \( \ell_\infty \)-norm (Guillaume et al., 1991). This is based on Pólya’s algorithm which states

\[ \lim_{p \to \infty} p_p = p_\infty \]  

where \( p = [\phi_2 \phi_3 \ldots \phi_n] \) and \( p_\infty \) is the minimax solution. This optimization process is initialized with the phases produced by the Schroeder-phase algorithm (Schroeder, 1970).
reactor is designed to heat a semiconductor wafer via radiation for chemical vapor deposition and other processing. The discrete MIMO description is presented as

\[ y_k = P_1 \cdot y_{k-1} + P_2 \cdot \left( \frac{y_{k-1}}{1000} \right)^4 + P_3 \cdot \left( \frac{y_{k-1}}{1000} \right)^3 + P_4 \cdot \left( \frac{y_{k-1}}{1000} \right)^2 + P_5 \cdot \left( \frac{u_{k-1}}{100} \right)^4 + P_6 \cdot \left( \frac{u_{k-1}}{100} \right)^3 + P_7 \cdot \left( \frac{u_{k-1}}{100} \right)^2 + P_8 \cdot u_{k-1} + P_9 \]  

This work is concerned only with SISO identification of the effects of lamp #2 on the temperature of location #2 on the wafer. The signals were generated based on a priori information and user decisions such that \( \tau_B = 33 \) sec, \( \tau_L = 7 \) secs, \( \alpha = 2 \), and \( \beta = 6 \) (Braun et al., 1999), which leads to \( n_s = 57 \) and \( N_s = 1246 \). The amplitudes of these signals were adjusted to provide equivalent power spectra. In Figure 2, note that the minimum crest factor approach improves the crest factor of the Schroeder signal with no high frequency content by 20% and by allowing some additional high frequency content the crest factor is improved by 31%. This is referred to as the “snow-effect” by (Guillaume et al., 1991). Figure 3 provides a look at the power spectra of the designs used in generating input for this example.

In Figure 4, the empirical transfer function estimate \( Y(e^{j\omega})/U(e^{j\omega}) \) is calculated using minimum crest factor signals without harmonic suppression and with even harmonics suppressed. Even harmonic suppression allows for a cleaner estimate of the linear kernel.

**4. CSTR CASE STUDY**

The CSTR model is based on a first principles analysis of a hypothetical CSTR (Bequette, 1998). The vessel is assumed to be perfectly mixed, and a single first-order exothermic, irreversible reaction, \( A \rightarrow B \), takes place. A diagram showing the vessel and the surrounding cooling jacket is presented in Figure 5. Based on accounting and conservation principles, the lumped parameter equations describing the system are

\[
\frac{dC_A}{dt} = \frac{F}{V} (C_{Aj} - C_A) - r
\]

\[
\frac{dT}{dt} = \frac{F}{V} (T_f - T) - \left( \frac{\Delta H}{c_p\rho} \right) r - \frac{UA}{c_p\rho V} (T - T_j)
\]

\[
r = k_o \exp \left( \frac{-\Delta E}{RT} \right) C_A
\]

This example is concerned with the Case 2 reactor parameter values presented by Bequette and shown in Table 1. The inlet stream is fed at a constant rate \( F \), with constant concentration \( C_{Aj} \) into the vessel. The final concentration of the reactant \( C_A \) is the controlled variable and the jacket temperature \( T_j \) is manipulated to keep the exit stream concentration \( C_A \) at setpoint. The exiting stream leaves at a rate \( F \) and since it is assumed the vessel is perfectly mixed, the exiting concent-
The input amplitude range chosen is ±25 K. This range is particularly interesting because the diabatic CSTR exhibits an “ignition” as the reactor settles from a +9 K or greater increase in jacket temperature.

The goal is to develop a nonlinear Model-on-Demand Model Predictive Controller (MoD-MPC) to regulate production about the high concentration, low temperature steady-state. To achieve this end, 4 multisine signals were designed and 1 m-level PRS signal was designed for excitation of the CSTR. For a discussion of MoD-MPC and design of m-level PRS signals, the user is referred to (Braun et al., 2000) in these proceedings. Signal designs of length beyond one day’s time were considered unacceptable. Previous attempts to formulate a linear PID controller revealed that a high bandwidth controller was needed to keep the reactor from “igniting.” Therefore, an α of 5 and a β of 1 were selected for design of all 5 signals. A sampling time of 0.1 hrs was used for all signals and measurements. For a switching time $T_{sw}$ of 0.7 hrs, which met the high frequency limit, only a 3-level PRS signal generated with 3 shift registers or a 5-level PRS signal generated with 2 shift registers meets the requirements. The 5-level is the signal of choice because 5-levels offers better resolution of the input space than does 3 and provides a crest factor of 1.39. The maximum move size of this signal is 50 K and the length was 16.8 hrs.

The various minimum crest factor signals evaluated are presented in Table 2. Note that one variation of a minimum crest factor signal is designed with exactly the relative frequency amplitudes as the m-level PRS. This provides a good basis for comparison and provides the multisine a near-white autocorrelation function. The low frequency limit dictated that the multisine signals should have a length of 12.6 hrs. Figures 6 and 7 contain the time domain realizations of all 5 input signals and the validation signal, and the power spectra of the input signals, respectively. From a “plant-friendliness” standpoint, the multisines are more attractive than the m-level PRS, mainly because the maximum step sizes are smaller than that for the m-level PRS. Another attractive feature is that the length of the multisine signal can be designed to precisely meet the low frequency limit.

The corresponding time domain realizations of the multisines with high frequency content and harmonic suppression provide better exploration of the interior of the input space, despite a lower crest factor. The validation signal excites the CSTR within the input range and provides power spectra and output well within the output limits of the 5 excitation signals. Figures 8 and 10 show
What can be concluded from the previous analysis is that, in a control relevant sense, multisine signals can produce an input/output database which is just as informative as that from m-level PRS excitation and the database is capable of supporting the MoD estimator. The main issues when choosing a multisine power spectrum are the uniformity and resolution the time-domain realization of that spectrum produces in the input/output space. For energy (TME) are included in the bottom plot of 11. The control experiment was subject to two ramp and one step setpoint changes at times 1, 5, and 15, respectively.

With a signal 4.2 hrs shorter, validation using the even suppression minimum crest factor multisines provides better RMS error and similar maximum error compared to the 5-level signal. For identification of nonlinear systems, little can be said relating the possible output range to crest factor for a given input range. However, as one considers linear identification or nonlinear identification of mildly nonlinear plants, a smaller crest factor can often provide a better signal to noise ratio in the output and provide a larger output range.

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Fig. 6. CSTR excitation and validation signals

Fig. 7. CSTR excitation power spectra

Fig. 8. CSTR output data

Fig. 9. CSTR database validation
5. CONCLUSION

We have shown in this paper that minimum crest factor signals can provide smaller move sizes and shorter identification times in comparison with m-level PRS signals while increasing resolution and uniformity in the input/output space of the plant under study. Harmonic suppression can be incorporated in the design of these signals to provide improved ETFE’s. These issues were demonstrated with a Rapid Thermal Processing reactor example and Model-on-Demand identification and control of an exothermic CSTR simulation.

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7. REFERENCES


